FRACTALCOMS

Exploring the limits of Fractal Electrodynamics for the future telecommunication technologies
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Task 1.2 Final Report “Fundamental Limits of Fractal Miniature Devices”

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Abstract:

The capability of fractal devices for reaching the fundamental limits established for Euclidean geometries is investigated. The existence of more realistic limits than the restrictive values found in the literature is also explored.

Keyword list: antenna, fractal, fundamental limit, efficiency, quality factor

FUNDAMENTAL LIMITS OF FRACTAL MINIATURE DEVICES

1 INTRODUCTION

Since the publication of [Puente, 1997], [Cohen, 1997], [Puente, 1998] and [Puente-Baliarda,2000] fractals deserved great attention. The general feeling about the seemingly unlimited potential of these geometries able to reduce their electrical size, and even able to approach their performance to a fundamental limit, was the source of such interest.

In the frame of the Fractalcoms Project, Task 1.2 is essentially addressed to the assessment of how effective is the occupancy of the volume inside the fictitious sphere that encloses a fractal device. This measure of effectiveness is evaluated with the help of a figure of merit called quality factor Q, that is related with the fractional bandwidth of the device.

The closeness of fractal devices to the fundamental limit established decades ago is also analysed and compared with standard Euclidean geometries to check the expectations put on these innovative designs.

Loss efficiency was also an important parameter to study in this task due to the theoretical infinite lengths that designs could reach and their high ohmic losses. Although practical applications pose a technological limitation in the iterative design procedure of these geometries (the intricacy of the shape of antennas and filters is limited), ohmic losses still play an important role on efficiency and insertion losses in these devices.

The existence of less restrictive values for the fundamental limit is explored. A more practical limit comes up after a multi-objective optimisation technique.

2 FUNDAMENTAL LIMITS AND FRACTALS

2.1 The Fundamental Limit

The performance of electromagnetic passive devices of Euclidean geometry is sensitive to its electrical size compared to the wavelength, i.e. given an operating wavelength and certain performance classical antennas and microwave devices can not be made arbitrarily small. There exist a fundamental law that restricts the performance of antennas fictitiously enclosed in a given volume. This law is stated in terms of a figure of merit called quality factor Q that is the ratio of time-average, non-propagating energy to radiated power of an antenna [Chu, 1948]

$$
\begin{aligned}
Q &= \frac{2\omega W_e}{P_{rad}} \qquad W_e > W_m \\
Q &= \frac{2\omega W_m}{P_{rad}} \qquad W_m > W_e
\end{aligned}
\tag{1}
$$

where P_{rad} is the radiated power, W_e and W_m are the stored electric and magnetic energy, respectively, and w is the radian frequency.

For linearly polarized antennas the fundamental limitation was found to be [McLean, '96]

$$
Q = \frac{1}{ka} + \frac{1}{(ka)^3} \tag{2}
$$

where k is the wave number at the operating wavelength and a is the radius of the smallest sphere that encloses the antenna (and its image in the case of monopoles). Similar expressions where found for circularly-polarized antennas.

In all the cases, reaching such an extreme quality factor requires that just a TE_{01} or TM_{01} spherical mode was excited (and a combined TE_{01}/TM_{01} mode for circularly polarization).

If the antenna is modelled as a resonant RLC circuit (series or parallel) the quality factor can be related with the fractional bandwidth B (normalised spread between half-power bandwidths) through

$$
Q \approx \frac{1}{B} = \frac{f_{center}}{f_{upper} - f_{lower}} \tag{3}
$$

This approximation is quite good for Q>>1 and it is rather inaccurate for Q<2, although useful to predict the potential broadband behaviour of the antenna. When Q>>1 the devices are narrow bandwidth and have large frequency sensitivity. Currents and ohmic losses in the antenna are large and there is a high reactive energy stored in its near zone.

2.2 Fractals

The scientific effort towards the miniaturization of communications systems also reached the antenna subsytem, and nowadays we are immersed in a continuous deployment of new antenna designs. Each is smaller than the previous one and even more integrated in the communications equipment.

A miniature antenna, or in other words an electrically small antenna, can be enclosed into a fictitious sphere with radius less than the radianlength ($a=\lambda/2\pi$), so that the product of the maximum antenna dimension times the wave number is less than the unity ($ka<1$). When working with electrically small antennas quality factors are high as seen from figure 1. This is the challenge of the antenna engineer, the design of miniature antennas capable of effective radiation on large bandwidths.

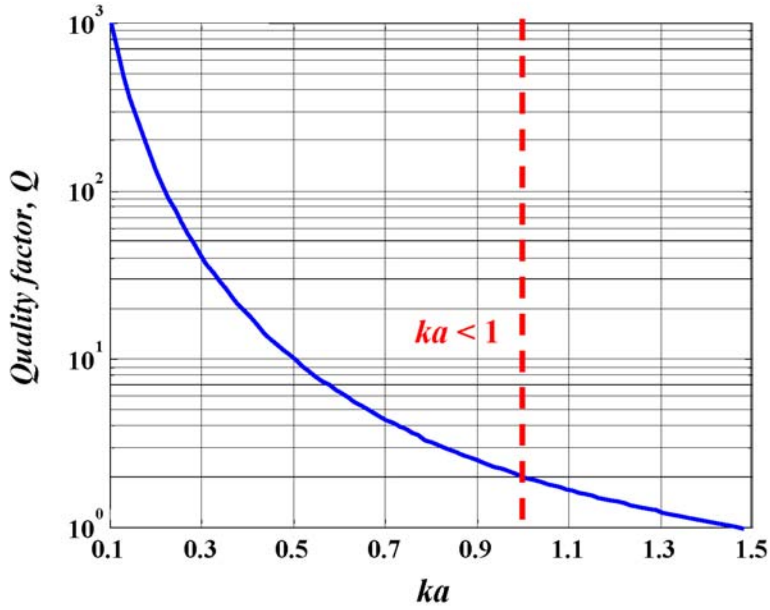

Figure 1. Fundamental limit of quality factor. The devices are electrically small when ka<1.

The use of fractal geometries to design new miniature devices has potential interest thanks to the inherent space-filling properties of some fractals, and also due to their fractional dimension, higher than the topological one. These geometries deserved special attention by the international scientific community since the publication of [Puente, 1997], [Cohen, 1997], and after the first experiences with these geometries. The increasing fractal iteration of a Koch monopole displayed a lower resonant frequency than the previous iteration and an smaller quality factor out of resonance.

Fractal geometries allow that infinitely large wires can be packed following deterministic rules into finite small spaces/volumes reducing the resonant frequency of the devices, i.e. reducing the electrical size of the antenna. Obvious technological limitations pose a constraint on the infinite number of iterations needed to build the fractal attractor. The iterative algorithm is stopped at the very beginning and consequently we have to work with a reduced-in-intricacy version of the fractal. This structure is called *pre-fractal*, and it is commonly accepted that it retains the properties of the fractal such as self-similarity and fractal dimension.

2.3 Radiation Pattern and Gain

Other parameters such as radiation pattern, directivity, gain, polarization and input impedance are important when designing an antenna. However, some of them do not vary much when dealing with electrically small antennas. In particular, small antennas in free space do have a *doughnut*-like radiation pattern with a directivity of 1.5 in linear scale.

As for antenna gain, the ohmic losses in the conductors have to be taken into account. Small antennas do usually have much losses and quite different loss efficiencies are expected according to their design. This parameter has been carefully considered in this task to evaluate the behaviour of pre-fractal antennas.

The input impedance of an small antenna is the sum of two factors: the radiation resistance of the antenna and its ohmic losses. Both are indirectly considered along this task due to their importance on the efficiency calculations, although not explicitly displayed because of the relative interest of their value. Nevertheless, we have to mention that the radiation resistance of an small antenna is always going to be lower than the radiation resistance of an small dipole with uniform current distribution, that is

$$R_{rad} = 80(ka)^2 \quad (4)$$

and will require the use of impedance matching networks or the connection to high impedance devices, according to applications.

Polarization will depend on the orientation of the electromagnetic fields radiated by the antenna and in the end it is controlled by the flow of electric or magnetic currents on the structure of the antenna. The importance of this parameter relies on the application the antenna is intended for.

3 FRACTAL DIMENSION OR TOPOLOGY

Results from task 1.1 “Understanding fractal electrodynamics phenomena” revealed that fractal dimension does not play a role in the expected improvement of the radiation performance of self-resonant pre-fractal wire monopoles. On the contrary, the use of pre-fractals with large fractal dimension for miniaturizing antennas provides worst performances than certain Euclidean designs. Although increasing the fractal dimension of antennas allows greater miniaturization ratios, poorer values of efficiencies and quality factors are achieved, obtaining unpractical designs for the vast majority of applications. Task 1.1 assessed that like Euclidean antennas, topology is what actually matters when designing an electrically small antenna.

These conclusions are the result from the measurements displayed in Figures 2, 3 and 4. Figure 2 summarizes the radiation performance of pre-fractal designs according to their fractal dimension and iteration. Their behaviour is compared with several straight monopoles loaded on their top with meander lines. The plot shows that better performances are achieved with the Euclidean monopoles for the same electrical size at resonance.

Figures 3 and 4 show the behaviour of several pre-fractals (with fractal dimension 1.58 and 2, respectively) generated with the same Iterative Function System (IFS) algorithm but with different initiators. Both figures agree in the fact that bending a continuous wire is suitable to reach greater miniaturization ratios. However, the radiation performance differs according to the topology of the structure (whether it is a bended wire structure or it contains loops) even when they have the same fractal dimension.

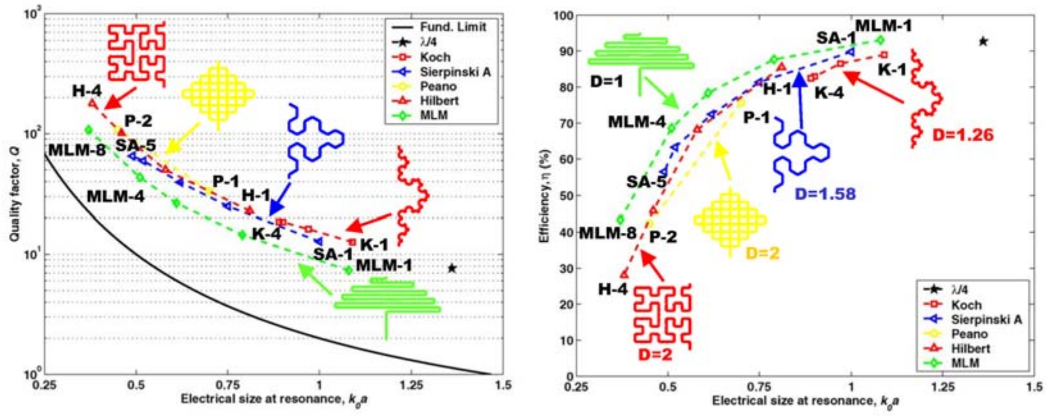

Figure 2. Measured quality factor and radiation efficiency maps of pre-fractals with different fractal dimension compared with meander line loaded monopoles.

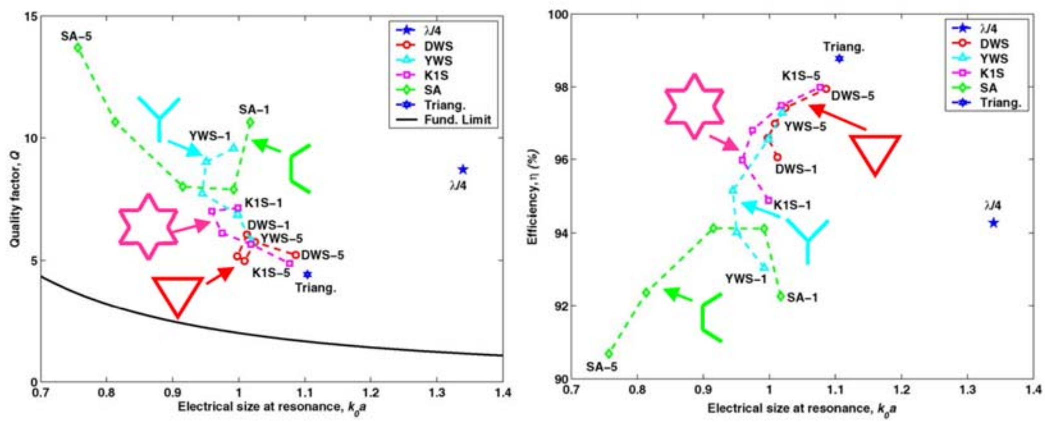

Figure 3. Measured quality factor and efficiency maps of pre-fractals "grown" with the Sierpinski gasket generator (D=1.58) and with different initiators (displayed in the figure).

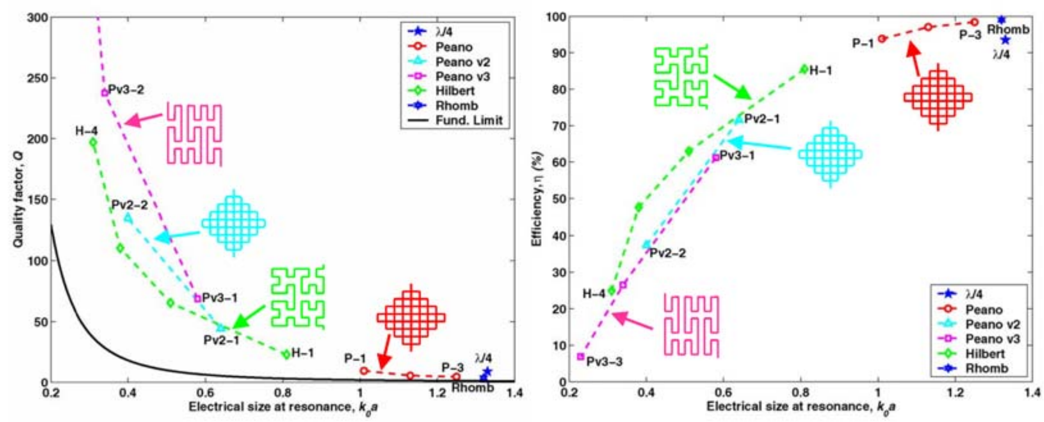

Figure 4. Measured quality factor and efficiency maps of pre-fractals "grown" with different IFSs. All the fractals have the same fractal dimension (D=2).

In the frame of this task we usually worked with planar designs. They are easily simulated and fabricated with standard techniques for manufacturing printed circuit boards for electronics circuits. Extending the previous conclusions to three dimensional pre-fractal designs should increase the effects of enlarging wire lengths and storing electromagnetic energy in the vicinity of the antenna. Therefore, efficiency and fractional bandwidth will decrease faster for these pre-fractal designs than for planar monopoles. These expectations have been proved after the analysis of a three dimensional Hilbert monopole (see Deliverable D1 “Task 1.1 Final Report”).

4 OTHER LIMITS FOR WIRE MONOPOLES

In previous sections we have seen that the fundamental limitation used in practice as a reference is far from the limit obtained with actual designs, either Euclidean or fractal. Therefore, it would be useful to determine more realistic limits for antennas. We have chosen to apply an heuristic mathematical method to obtain extrema and to asses whether Euclidean geometries are capable of combining miniaturization and closeness to the fundamental limit. A optimisation technique known as genetic algorithms (GA) [Back, 1997] has been implemented.

In few words, GAs are methods for seeking extrema of any function $f(x)$, where x, known as an individual, is a codified form (real or binary) of the choosing parameters in the optimisation process. As a measurement of the optimisation a fitness function is needed. GA starts by randomly generating a initial population of individuals that are classified according to their fitness function values. Genetic operators such as selection, crossover and mutation, are subsequently applied so the generations of individuals evolve towards ones that give optimised values of the requested parameter.

The same optimisation schemes can be used for more complicated optimisations, as those cases when several parameters are required to be optimised [Back, 1997]. The main difference in this case is the multi-objective operators used to determine how good each individual is and the selection process after each generation of individuals is built. It is worth to point out that in these cases the final solution of the problem is not unique, and it forms an optimal front of solutions, which mixes individuals with good characteristics in each optimised parameter and individuals with a good compromise between them.

In particular we wanted to obtained a more realistic practical limit for the Q value of small antennas and to compare it with the theoretical one given by Chu. With this aim a multi-objective Genetic Algorithm (GA) tool has been applied in conjunction with the numerical electromagnetic code (NEC) to the optimisation of wire antennas in terms of the Q factor, but having at the same time a small electrical size and high efficiency.

A flowchart that summarizes the way our GA multi-objective code works is given in Figure 5. It starts by a randomly generated initial population of NPOP individuals which in our case are thin-wire antennas, either zigzag type or meander type, that fits inside a circumference of radius a (the monopole antennas utilize half the circle). Figure 6 shows examples of a meander-type and a zigzag type antenna. The NEC code is applied to solve, in the frequency domain, the electric field integral equation formulated for the currents induced along the wire segments. Once the current is calculated several other parameters are obtained as the Q factor, the resonance frequency and the efficiency and three fitness functions are defined that rewards individual with low Q, high efficiency and low resonance frequency. The individuals are classified according to the value of those three fitness function (using the ranking operator) and also the sharing operator is used, which penalized individuals that are too similar to others. The subsequent application of the GA-operators (tournament-method selection, one-point crossover and a Gaussian-probability-distribution mutation) produces the next generation. Of course, in each generation we need to check out if the optimisation is good enough to end the process. As we mentioned before, the final optimised solution is not an unique individual, the multi-objective GA renders a front or surface of optimal solutions (Pareto front) from which the designer can select the individual that best fits the requirements of the problem at hand.

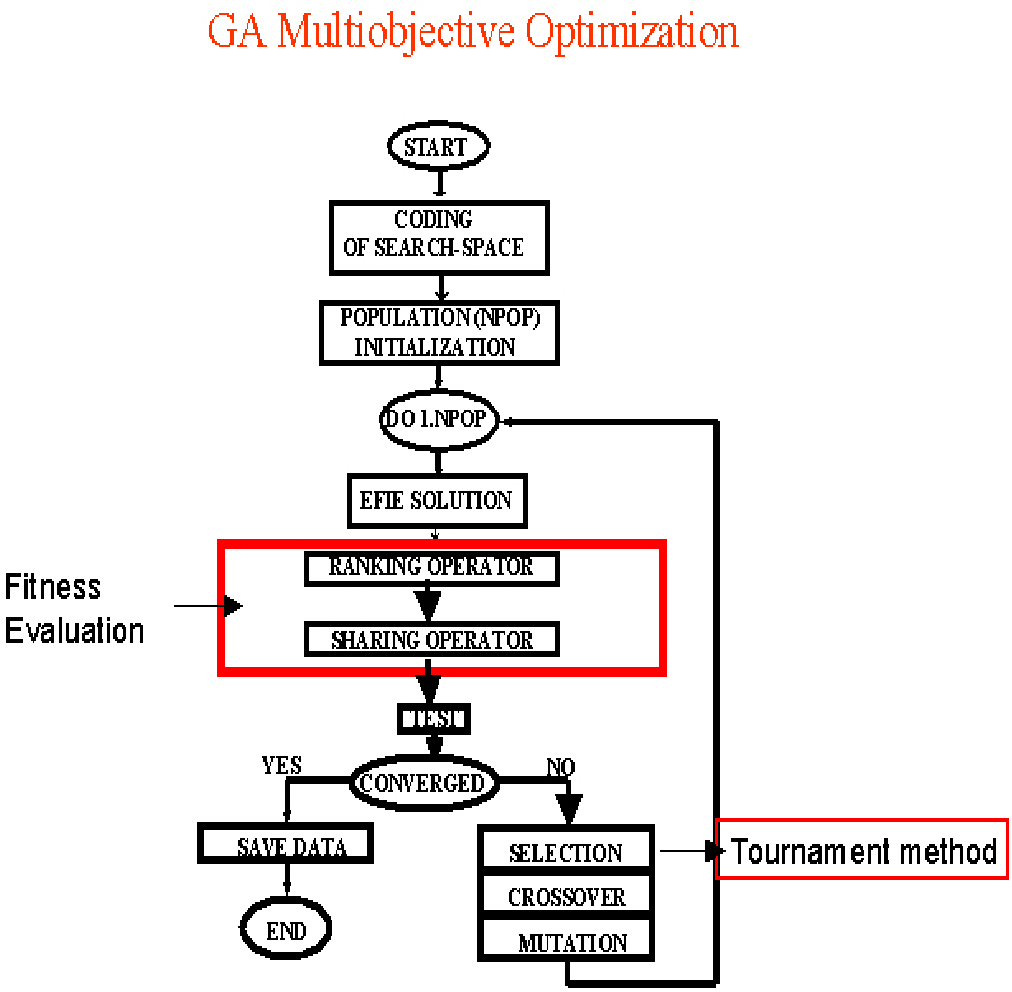

Figure 5. Flowchart of the Genetic Algorithm Multi-objective Optimisation.

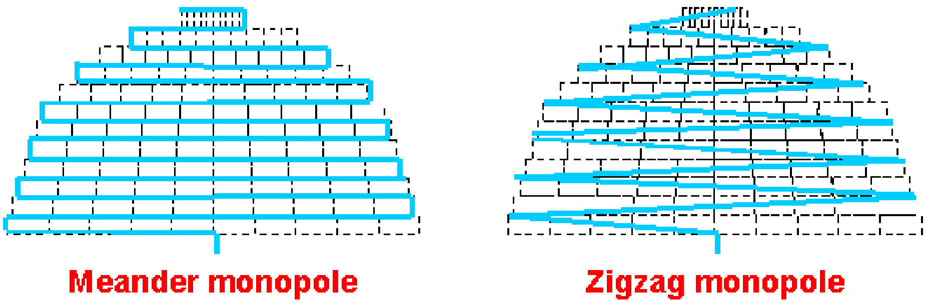

Figure 6. Example of meander and zigzag type antennas.

Planar meander line and zigzag type monopoles (restricted to 12 wire segments) inscribed into an hemisphere of fixed radius have been designed using the previously described multi-objective optimisation technique based on GA. Electrical size at resonance, efficiency and quality factor are the parameters that manage the evolution of the algorithm. Both types of monopoles are chosen because of their checked suitability to fabricate miniature antennas. After the optimisation procedure, a set of optimum solutions (that survived the evolutionary procedure of optimisation) have been found by the algorithm. The Pareto front of this set of solutions is a surface that is displayed in Figure 7, where the expected performance of each design is shown in terms of SWR versus the resonant frequency and the wire length of each antenna. In order to facilitate the visualization and understanding of the results we just plot, in Figure 8, the efficiency and Q factor of a set of individuals selected from the Pareto surfaces. In particular the individual with a lower Q has been chosen.

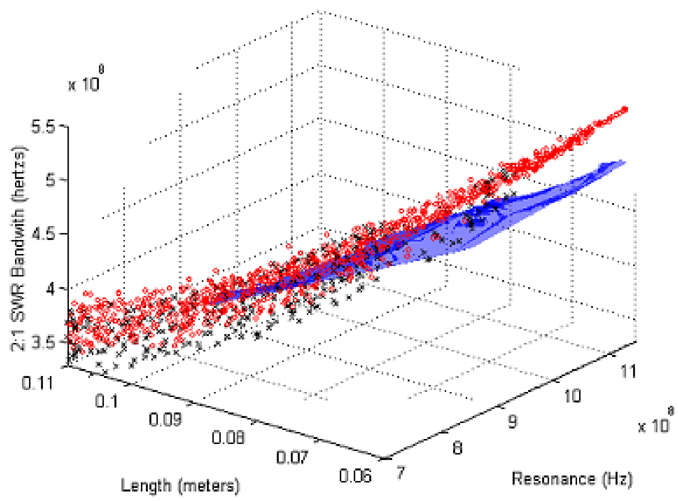

Figure 7. Pareto front of a multi-objective optimisation.

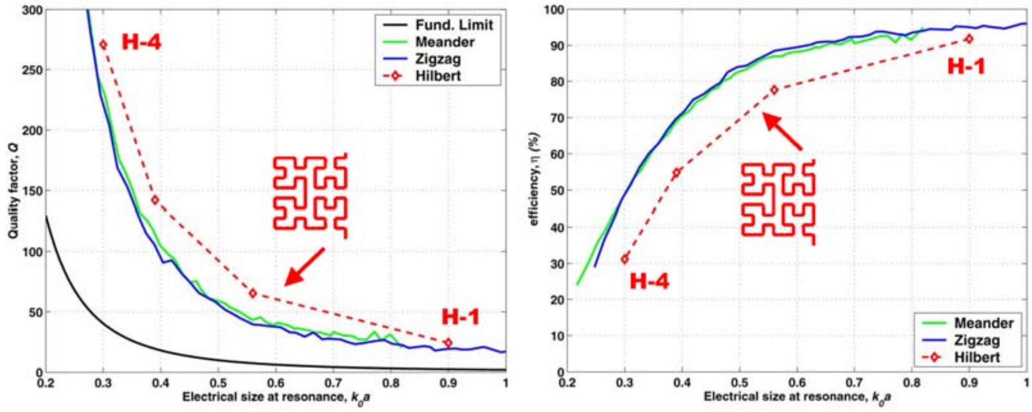

Figure 8. Quality factor (left) and efficiency (right) plots for a set of individuals with lower Q extracted from the Pareto front. Both are compared with the performance of a planar Hilbert design.

Figures 7 and 8 provide a practical limit more realistic than Chu's for the design of small antennas that fits in the half circle of radius a and in the range of frequencies analysed. It should be pointed out that all the GA designs are for wire structures that lay on a plane and that present no loops in their geometries. As future work, 3D geometries and structures with loops should be considered which we think will improve the practical limits given here.

The results show that the miniaturization of the antenna (the reduction of its electrical size *ka*) is translated into a worsening its radiation performance: efficiency and fractional bandwidth decrease reaching unpractical values for the vast majority of common applications.

In conclusion, and as expected for electrically small antennas, although their radiation pattern and directivity remains unchanged, the loss efficiency of the structures reduces the gain of the designs and their fractional bandwidth is also reduced. This behaviour, represents the best performance that a planar self-resonant wire monopole can reach. Figure 8 also included a plot of the performance of a Hilbert planar monopole for iterations 1 to 4 to show their closeness to the practical limits for efficiency and quality factor.

All the results displayed in the plots have been computed for designs with the same wire radius. Slightly more efficient designs are expected by increasing the wire radius of the antennas. Obviously this improvement is expected both for the Euclidean and the pre-fractal monopoles.

5 STATE OF THE ART

Recently, in June 2003, it has been published a paper [Thiele, 2003] where a more realistic lower bound for the fundamental limit on the radiation *Q* has been estimated.

This prediction is based in the assumption of a sinusoidal current distribution along an electrically small antenna, in contrast with the classical approach where an Hertzian dipole with a uniform current distribution was considered.

Under this consideration, the lower limit for the radiation Q is evaluated from the expression

$$Q = \frac{\int_{-\infty}^{-\pi L/\lambda} \left|E_n(u)\right|^2 du + \int_{\pi L/\lambda}^{\infty} \left|E_n(u)\right|^2 du}{\int_{-\pi L/\lambda}^{\pi L/\lambda} \left|E_n(u)\right|^2 du} = \frac{\int_{\pi L/\lambda}^{\infty} \left|E_n(u)\right|^2 du}{\int_{0}^{\pi L/\lambda} \left|E_n(u)\right|^2 du} \tag{5}$$

being $E_n(u)$ the normalised field pattern radiated by the antenna, a linear source of length L along the z axis. The field is represented in the u-space, where u is $u=(\beta L/2)\cos\theta$, being β the wavenumber and θ the direction from the z axis to the field point.

Authors of [Thiele, 2003] found this realistic limit through the consideration of a dipole with a sinusoidal current distribution, that generates a normalised field pattern that follows equation (6)

$$E_n(u) = \frac{\cos u - \cos\left(\frac{\beta L}{2}\right)}{\left[1 - \cos\left(\frac{\beta L}{2}\right)\right]\sqrt{1 - \left(\frac{2u}{\beta L}\right)^2}} \tag{6}$$

The limit calculated from the far field pattern is shown in Figure 9 with the classical fundamental limit. Both are compared with the measured radiation Q of the pre-fractal monopoles of Figure 2 and the computed radiation Q of the optimised planar designs found using genetic algorithms (see Section 4). Highly iterated pre-fractals have a current distribution different from a sinusoidal one along the z axis, being more uniform at the first segments of the monopoles. This is the explanation why the pre-fractal monopoles are under the far field Q limit and closer to the classical fundamental limit. For low iterated pre-fractal monopoles the current distribution is closer to a dipole and their limit is the limit computed using (5) and (6).

The same behaviour is more notorious in the case of meander line loaded monopoles and the genetically optimised planar monopoles. Small monopoles have a more uniform current distribution in their first segments while the rest of the structure act as a capacitive load. Their limit is the classical limit established by Chu and McLean. When the monopoles become electrically larger ($ka \rightarrow 1$) a sinusoidal current distribution along a wire with an infinitesimal radius is the limit.

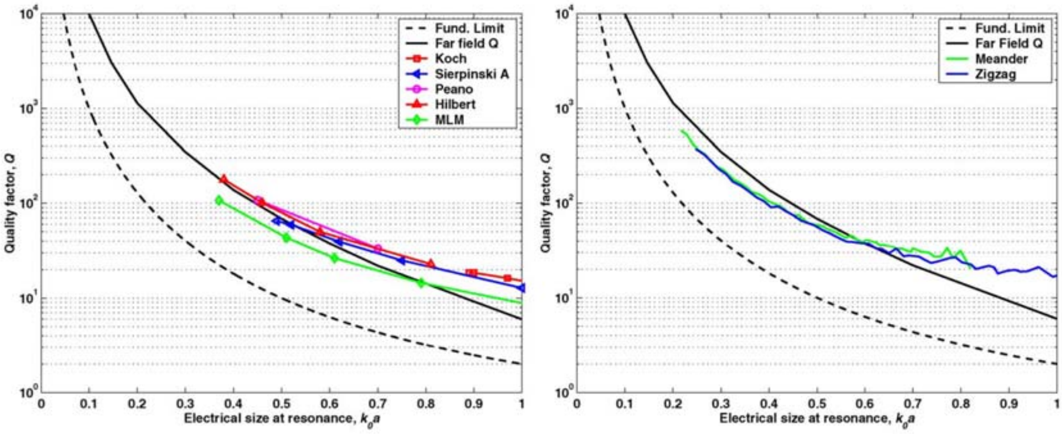

Figure 9. Fundamental limit of radiation Q for a sinusoidal current distribution (black continuous line) along a wire. This limit is compared with the classical limit by Chu and McLean (black dashed line). Both limits are compared with measured lossless Q of pre-fractal monopoles of different fractal dimension (left plot) and computed Q of genetically designed planar monopoles (right plot).

6 GUIDELINE FOR THE DESIGN OF ELECTRICALLY SMALL ANTENNAS

From the evolution of task 1.2 it has been assessed through simulations and measurements that a suitable design that gets closer to the classical antenna limit on the radiation quality factor *Q* requires a uniform current distribution along the antenna (in fact this limit was established for an Hertzian dipole). In this sense top loading is needed to allow charge storage.

An effective design would increase the bandwidth of the antenna without increasing its electrical size. Our future line of research is the use of fractal loading (using three dimensional pre-fractals, planar pre-fractal loading or pre-fractal surfaces) in comparison with Euclidean loading.

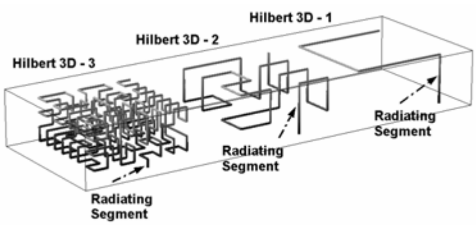

Figure 10. Example of pre-fractal loading using 3D pre-fractals.

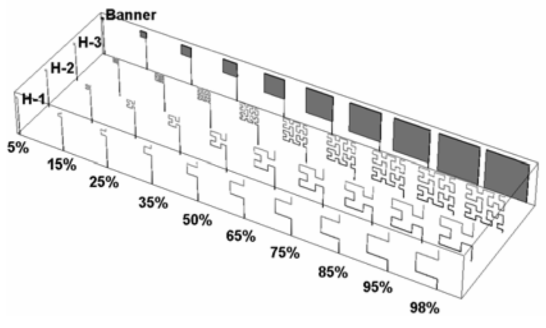

Figure 11. Example of pre-fractal loading using a planar pre-fractal.

7 CONCLUSION

Several conclusions are extracted from the research developed in the frame of this project in relation with the miniaturization of devices using fractal (pre-fractal) geometries.

- The reduction on the resonant frequency of a pre-fractal device with the increasing iteration tends to a limiting value due to the “shortcut effect” (see Deliverable D1 “Task 1.1 Final Reort”). This effect poses a limit on the degree of miniaturization that a pre-fractal could reach (without taking into consideration the technological limitations in the manufacturing procedures) and on the radiation performance of such a structure.

- Pre-fractal geometries are suitable designs to fit long wires inside small areas/volumes using compact algorithms (IFS). This capability is of potential interest to design miniature antennas, i.e. electrically small antennas, thanks to the reductions in the resonant frequency attained when increasing the pre-fractal iteration and the fractal dimension of the geometries. However, a swift decrease on efficiency and fractional bandwidths (inverse of quality factor) are also observed. Other Euclidean designs show better performances than pre-fractals for the same miniaturization ratios.

- Some pre-fractal designs that have an Euclidean counterpart perform quite similarly than their counterpart but with smaller electrical sizes. Nevertheless, this behaviour is exclusive for certain designs.

- It has been empirically assessed that the fundamental limit of radiation quality factor of antennas holds even for pre-fractal devices. Moreover, pre-fractals are not closer to this borderline than other standard Euclidean designs.

- A multi-objective optimisation technique based on Genetic Algorithms assessed the existence of practical limits less restrictive than the fundamental limit predicted by Chu and reviewed by McLean. Both Euclidean and pre-fractal geometries are near this practical limit. The limit has been found for planar self-resonant wire monopoles. A closer practical limit could be found if three dimensional self-resonant monopoles would have been analysed.

- A more realistic limit that holds for antennas with sinusoidal current distribution has been recently published [Thiele, 2003]. It assessed that end-loaded antennas (with Euclidean or pre-fractal loads) achieve better performances than small antennas with sinusoidal current distributions.

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